2. Spatial Concepts and Representation of Spatial Objects

Overview

Learning Objectives

- Understand fundamental properties of space concepts
- Learn about models and techniques for space modeling
- Understand models and techniques for presenting spatial objects and collections of objects
- Learn about spatial abstract data types (spatial ADTs)

Literature

- [RSV02] Chapter 2
- [SC03] Chapter 2
- Markus Schneider: *"Spatial Data Types: Conceptual Foundation for the Design and Implementation of Spatial Database Systems and GIS",* tutorial given at 6th International Symposium on Advances in Spatial Databases (SSD'99), 1999.
  http://www.cise.ufl.edu/~mschneid/Service/Tutorials/TutorialSDT.html

[Disclaimer: Some figures on the following slides are taken from wikipedia.org]

Topology

Topology

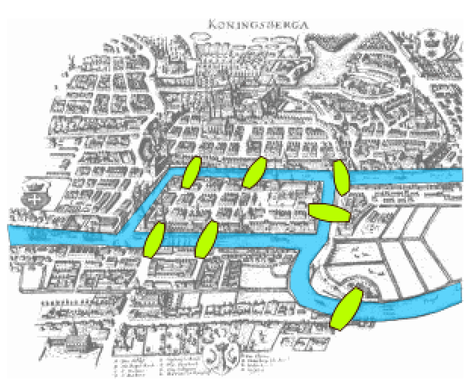
- Branch of mathematics that deals with spatial relationships that are preserved under *bicontinuous deformation* (stretching without tearing or gluing), sometimes called *rubber-sheet geometry.*
- The idea is that some geometric problems do not depend on the exact shape of objects involved, but rather on the way objects are "connected to each other"

- Spaces studied in topology are called *topological spaces*. ☞
- Elementary topology is often called *point-set topology.*
- Two spaces are topologically equivalent if one can be transformed into the other without cutting it apart or gluing pieces of it together.

  Spaces are then *homeomorphic* (continuous bijection with a continuous inverse).

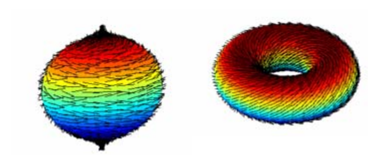
Topology (2)

Seven Bridges of Koenigsberg



Leonhard Euler (1736):
It is impossible to find a route through the town of Koenigsberg that would cross each of its seven bridges exactly once.

Others, e.g., the hairy ball theorem



Topological Space

Topological spaces are structures that allow one to formalize concepts such as convergence, connectedness, and continuity.

<u>(Def)</u> A **topological space** is a set *X* of elements, called *points*, with a collection *T* of subsets of *X,* called *open sets,* that satisfy the following three axioms:

[A1] The empty set and *X* are in *T*.
[A2] The union of any collection of sets in *T* is also in *T*.
[A3] The intersection of any pair of sets in *T* is also in *T*.

When, for a given point set *X*, we devise a system of subsets of *X* that satisfy A1-A3, we say that *X* has received a **topology**.

If *T* contains all subsets of *X,* then a *discrete topology* is imposed on *X.*

The set of real numbers **R** is a topological space: open sets are generated by the base of open intervals.

Topological Space (2)

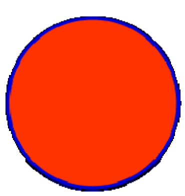
Topological property (or topological invariant):

Property of a toplogical space which is invariant under homeomorphisms; such a property can be expressed using open sets

E.g., a space *X* is connected if it is not the union of a pair of disjoint non-empty *open sets*.

Open set: a set *U* is called open if starting from any point in *U* one can move by a small amount in any direction and still be in *U*. I.e., the distance between any point $x \in U$ and the edge of *U* is always > 0.

Example:



Metric Space

A metric space is a set where the notion of *distance* between elements of the set is defined. A typical example is the 3D Euclidean space. A metric space induces topological properties like open and closed sets.

(Def) A **metric space** is a 2-tuple (X,d) where X is a set and d is a metric on X, i.e., a function $d: X \times X \rightarrow \mathbf{R}$ such that

1. $d(x,y) \geq 0$ *(non-negative)*
2. $d(x,y) = 0$ iff $x=y$ *(identity of indiscernibles)*
3. $d(x,y) = d(y,x)$ *(symmetry)*
4. $d(x,z) \leq d(x,y) + d(y,z)$ *(triangle inequality)*

Examples:

real numbers with distance function $d(x,y) = |y-x|$ or more generally Euclidean n-space with Euclidean distance;

any normed vector space; Manhattan norm; Chessboard distance;

for systems of roads and terrains, the distance between two locations can be defined as the length of the shortest route;...

Metric Space (2)

In any metric space M one can define the **open balls** as the sets of the form

$$B(x,r) := \{y \in M : d(x,y) < r\} \text{ with } x \in M \text{ and } r \in \mathbf{R}$$

A subset of M that is the union of open balls is called an open set.

Every metric space is automatically a topological space, the topology being the set of all open sets. $\Rightarrow$ *metrizable space*

- A metric space is *bounded* if $\exists\, r : d(x,y) \leq r$ for all x,y in M; the smallest such r is called *diameter* of M.
- For $S \subseteq M$ and $x \in M$, the *distance* from x to S is defined as $d(x,S) := \inf\{d(x,s) : s \in S\}$

Also, to preserve at least the topological structure induced by a metric, it requires at least the existence of a continuous function between them (morphism preserving the topology of the metric spaces).

Euclidean Space

- Generalization of 2D and 3D spaces studied by Euclid
- Applies concepts such as length and angle to a coordinate systems in any number of dimensions
- Standard example of a finite-dimensional, real, inner product space
- Particular metric space that enables investigation of topological properties

(Def) For $n \in$ **N**, the space of all n-tuples of real numbers forms an n-dimensional vector space over **R**, called the **real coordinate space**, denoted $\mathbf{R}^n$.

- element x $\in \mathbf{R}^n$ is written $x = (x_1, x_2, ..., x_n)$
- standard vector operations apply (addition, scalar multiplication etc.)
- $\mathbf{R}^n$ comes with standard basis (unit vectors)

Euclidean Space (2)

The Euclidean space is more than just a coordinate space. In order to be able to talk about distance between points and angles between lines or vectors, one uses the *inner product* (also called *dot product*) on $\mathbf{R}^n$:

$$\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n.$$

For any two vectors *x* and *y*, dot product gives a real number; this allows us to define the "length" of a vector (also called *Euclidean norm* on $\mathbf{R}^n$):

$$\|\mathbf{x}\| = \sqrt{\mathbf{x} \cdot \mathbf{x}} = \sqrt{\sum_{i=1}^{n} (x_i)^2}$$

One can use the norm to define the *Euclidean metric*:

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}.$$

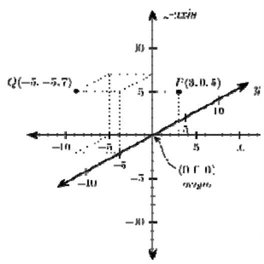
Real coordinate space together with above Euclidean structure is called **Euclidean Space**, denote $\mathbf{E}^n$.

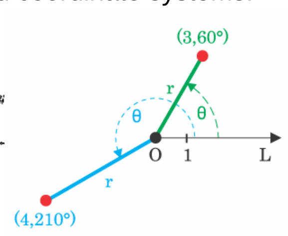
Coordinate Systems

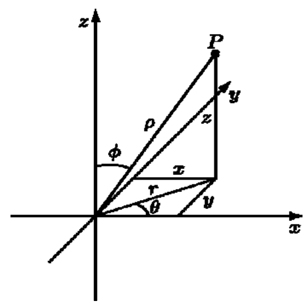
A **coordinate system** assigns an *n*-tuple of numbers to each point in an *n*-dimensional space.

A *coordinate transformation* is a conversion from one system to another, to describe the same space.

Some commonly used coordinate systems:







3D Cartesian system | Polar coordinates | Spherical coordinate system

Geographic Space Modeling

There are two common types of models for spatial information:

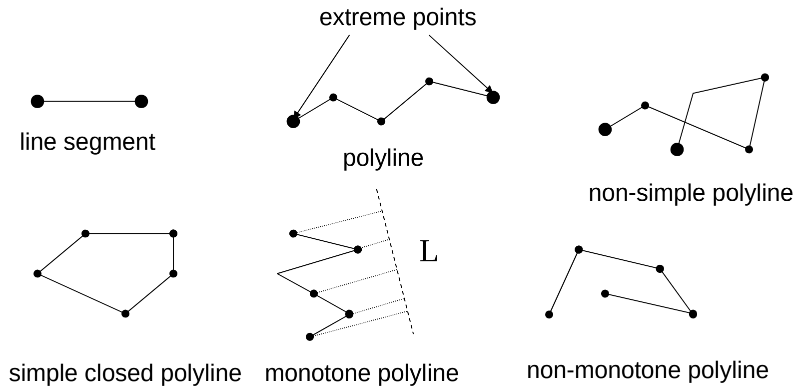
- Entity-based
- Field-based

In the entity-based model, geographic/spatial objects correspond to (real-world) features about which information has to be recorded. In practice, one uses the following types of spatial objects:

1. *0-dimensional objects* or *points*
   Used to represent location of entities whose shape is not considered useful or essential; note that shape depends on the (map) scale!
2. *1-dimensional objects* or *linear objects*
   Commonly used to represent networks (roads, utility lines etc).
   The basic spatial type considered in the following is the *polyline.*

Polyline

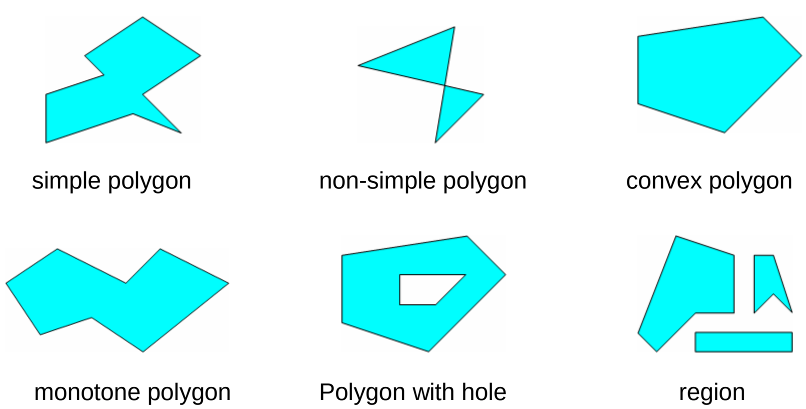
(Def) A **polyline** is a finite set of *line segments* or *edges*, such that each segment endpoint (called *vertex*) is shared by exactly two segments, except for the two endpoints (called *extreme points*) that belong to only one segment.



Polygon

3. *2-dimensional objects* or *surfacic objects*

   Mostly used to represent large areas. *Polygons* are the main spatial type for such objects. A polygon is a region of the plane bounded by a closed polyline, called its *boundary*.



Remarks on Entity-based Model

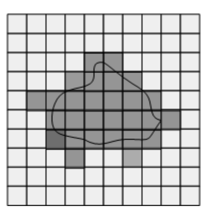
- Choice of geometric type is arbitrary and depends on future use of spatial object in applications.
- Choice may be influenced by scale of interest for intended application.
- It might be useful to store different representations of the same entity at various scales to derive proper representation on demand.
- Because only line segments are used, linear and surfacic objects are only a linear approximation of entities and could be presented more precisely through higher-order polynomials.
- There is always a tradeoff between faithful approximation and the number of segments for representing a curve.

Scale or geographic detail is an essential property of every GIS project!

Field-based Models

There are three main concepts in field-based models:

1. The underlying (geographic) space is partitioned (tessellated) into cells.
2. With each point in space one or more attribute values are associated, defined as *continuous functions* in *x* and *y* (for 2-dimensional space).
   The measure of several phenomena can be collected as attribute values varying with location in the space.
3. The concept of object is not relevant in the field-based approach.



There is a dictum of
classical physics
that states that
in nature everything is continuous.